$$x_1 = \sigma \sqrt{A} \operatorname{dn} w_1 \left\{ \frac{1}{A\sigma^3 \varkappa'^2} \left[\mathrm{E}(\operatorname{am} w_1) - \frac{\varkappa^2 \operatorname{sn} w_1 \operatorname{cn} w_1}{\operatorname{dn} w_1} \right] \beta_1 + \beta_2 + \ldots \right\} \tag{8}$$

$$y_1 = \sigma \sqrt{A} \varkappa' \frac{\operatorname{sn} w_2}{\operatorname{cn} w_2} \left\{ -\frac{1}{A\sigma^3 \varkappa'^2} \left[\frac{\operatorname{cn} w_2 \operatorname{dn} w_2}{\operatorname{sn} w_2} + \mathrm{E}(\operatorname{am} w_2) \right] \beta_3 + \beta_4 + \ldots \right\}$$

Using (8) we reduce conditions (7) of existence of periodic solutions to the inequalities

$$\left[\frac{D(\psi_1, \psi_2)}{D(\beta_1, \beta_2)} \right]_{k=0} = -\frac{2}{\sigma \varkappa'^2} \mathrm{E}(\varkappa) \neq 0, \quad \left[\frac{D(\psi_3, \psi_4)}{D(\beta_3, \beta_4)} \right]_{k=0} = 2E(\varkappa)$$

which are satisfied for any periodic solutions escept for $\varkappa' = 0$.

Quasi-periodic motions of the solid body generally correspond to the derived periodic solutions of system (1).

In fact let us consider the cyclic integral

$$\psi' = \frac{If - C\varphi' \cos \vartheta}{(A \sin^2 \varphi + B \cos^2 \varphi) + C \cos^2 \vartheta}$$

By expanding its right-hand part in Fourier series in multiples of argument τ / T and integrating, we obtain $\psi = n(\tau - \tau_0) + \Phi(\tau)$, where $\Phi(\tau + T) = \Phi(\tau)$, and n is a constant quantity dependent on initial conditions. It is obvious that generally nT is not a multiple of 2π, which shows the validity of the above conclusion.

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ON THE STRUCTURE OF FORCES

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It is shown that a wide class of nonlinear forces can be represented by the sum of potential, nonconservative position, gyroscopic and dissipative forces.

Investigation of motion stability is in many instances conveniently achieved by analyzing the structure of acting forces. This method, whose basis was established in [1], has been recently successfully applied mainly to linear systems of arbitrary forces which can be fully represented as the sum of potential, nonconservative position, gyroscopic and dissipative forces. It is shown in this paper that such representation of forces can also be applied to a wide class of nonlinear forces.

Let an arbitrary vector field $\mathbf{Q}(x) = \mathbf{Q}(x_1, \ldots, x_n)$ be specified in some region of an n-dimensional orthogonal space $x = (x_1, \ldots, x_n)$. We call the vector field $\mathbf{R}(\mathbf{x})$ a circulation field, if at every point M of the specified field region vectors $\mathbf{R}$ and $\mathbf{x}$

are normal to each other ($\mathbf{x}$ is the radius vector of point M)

$$\mathbf{R}\cdot\mathbf{x} = \sum_{j=1}^{n} R_j x_j = 0 \tag{1}$$

Theorem. Any arbitrary vector field $\mathbf{Q}(x)$ that is continuous together with its first order derivatives can be always resolved into potential and circulation fields

$$\mathbf{Q}(x) = -\operatorname{grad}\Pi + \mathbf{R}(x) \tag{2}$$

in which the field $\mathbf{R}(x)$ and the potential Π are to be determined.

Proof. Components K_j of the potential field $\mathbf{K}(x) = -\operatorname{grad}\Pi$ satisfy conditions

$$\frac{\partial K_j}{\partial x_k} = \frac{\partial K_k}{\partial x_j} \quad (k, j = 1, \ldots, n) \tag{3}$$

The scalar form of equality (2) is

$$Q_k = K_k + R_k \quad (k = 1, \ldots, n) \tag{4}$$

With this formulas (3) reduce to the form

$$\frac{\partial R_j}{\partial x_k} = \frac{\partial R_k}{\partial x_j} - \Phi_{kj}, \ \Phi_{kj} = \frac{\partial Q_k}{\partial x_j} - \frac{\partial Q_j}{\partial x_k} \tag{5}$$

If all $\Phi_{kj} = 0$, the field $\mathbf{Q}(x)$ is potential and $\mathbf{R} = 0$.
Partial differentiation of equality (1) with respect to x_k yields

$$\sum_{j=1}^{n} x_j \frac{\partial R_j}{\partial x_k} = -R_k \quad (k = 1, \ldots, n)$$

Using equality (5) we obtain

$$\sum_{j=1}^{n} x_j \frac{\partial R_k}{\partial x_j} = H_k - R_k, \quad H_k = \sum_{j=1}^{n} \Phi_{kj} x_j \quad (k = 1, \ldots, n) \tag{6}$$

Thus components R_k satisfy n nonhomogeneous linear differential equations in partial derivatives, in which functions $H_k(x)$ are known. Let us compose a system of ordinary differential equations which would correspond to Eq. (6), and determine n first integrals of that system

$$x_1 = C_1 x_n, \ldots, x_{n-1} = C_{n-1} x_n, \quad R_k = \frac{1}{x_n} \int^{x_n} H_k \, dx_n + \frac{C_n}{x_n}$$

where C_j are constants of integration.

The general solution of Eq. (6) can be represented in the form

$$R_k = \frac{1}{x_n} \int^{x_n} H_k \, dx_n + \frac{1}{x_n} \Psi_k \left(\frac{x_1}{x_n}, \ldots, \frac{x_{n-1}}{x_n} \right) \quad (k = 1, \ldots, n)$$

where Ψ_k is an arbitrary function.

For $H_k = 0$ $(k = 1, \ldots, n)$ all $R_k = 0$ (see the remark to formula (5) and equalities (6)). Hence, setting $\Psi_k = 0$, for the components of the nonconservative position force we obtain

$$R_k = \frac{1}{x_n} \int^{x_n} H_k (C_1 x_n, \ldots, C_{n-1} x_n, x_n)\, dx_n \quad (k = 1, \ldots, n) \tag{7}$$

in which, after squaring x_j / x_n $(j = 1, \ldots, n - 1)$ are to be substituted for all C_j.

Components of the potential field $\mathbf{K}(x)$ are determined by the equality (4), after which the field potential $\Pi(x)$ is determined in the usual manner.

Note that for linear fields $\mathbf{Q} = C\mathbf{x}$, where C is an arbitrary square matrix of order $n \times n$, the proof is elementary: it is sufficient to resolve matrix C into a symmetric and a skew-symmetric parts.

Example. Let a vector field be specified by its components

$$Q_1 = 2x_1^{11/3} - x_1^{4/3} x_2, \quad Q_2 = -x_1^{7/3} + x_2$$

Using the proposed method we determine the components of the circulation and potential fields

$$R_1 = {}^2/_5 x_1^{4/3} x_2, \quad R_2 = -{}^2/_5 x_1^{7/3}, \quad \Pi = -{}^3/_7 x_1^{14/3} + {}^3/_5 x_1^{7/3} x_2 - {}^1/_2 x_2^2$$

Let us apply the theorem proved above to resolving forces into components. We determine the system position by n generalized coordinates $q_1, \ldots, q_n$, and introduce in the investigation an n-dimensional orthogonal space $(q_1, \ldots, q_n)$ and two vectors: $\mathbf{q} = (q_1, \ldots, q_n)$ and $\mathbf{Q} = (Q_1, \ldots, Q_n)$, where Q_k are the generalized forces of the system. The first of these vectors is assumed to define a representative point and the second the force applied at that point.

In the linear theory, force $\mathbf{R} = -P\mathbf{q}$, where P is a skew-symmetric matrix of order $n \times n$, is called the nonconservative positional force (in Ziegler's terminology [2] it is called the circulation force). This force is normal to $\mathbf{q}$, since $\mathbf{R} \cdot \mathbf{q} = -P\mathbf{q} \cdot \mathbf{q} \equiv 0$. We shall generally call nonconservative position force any force $\mathbf{R}$, if it is normal to $\mathbf{q}$

$$\mathbf{R} \cdot \mathbf{q} = \sum_{j=1}^{n} R_j q_j = 0 \tag{8}$$

On the basis of the above theorem it is possible to resolve any position force $\mathbf{Q}^*(q)$ that is continuous with its first order derivatives into potential and nonconservative position components

$$\mathbf{Q}^*(q) = -\operatorname{grad} \Pi + \mathbf{R}(q) \tag{9}$$

where Π is the potential energy of the system.

Let us now consider forces $\mathbf{Q}^{**}(q^\cdot)$ which depend on velocity. In conformity with the definition given by Thomson and Tait [1], force $\mathbf{\Gamma}$ is called gyroscopic if its power is zero

$$\mathbf{\Gamma} \cdot \mathbf{q}^\cdot = \sum_{j=1}^{n} \Gamma_j q_j^\cdot = 0 \tag{10}$$

In the velocity space $(q_1^\cdot, \ldots, q_n^\cdot)$ the gyroscopic force has the property of orthogonality (1). Hence force $\mathbf{Q}^{**}(q^\cdot)$ can be resolved into two components

$$\mathbf{Q}^{**}(q^\cdot) = -\operatorname{grad} F + \mathbf{\Gamma}(q^\cdot)$$

Force

$$\mathbf{D}(q^\cdot) = -\operatorname{grad} F, \quad D_k = -\partial F / \partial q_k^\cdot \quad (k = 1, \ldots, n)$$

is dissipative with positive or negative resistance. On the basis of the last equalities function F may be called the generalized Rayleigh function. (Another generalization of the Rayleigh function is due to Lur'e [3]).

Thus the following theorem is proved.

T h e o r e m. The arbitrary force

$$Q(q, q^{\cdot}) = Q^*(q) + Q^{**}(q^{\cdot})$$

that is continuous with its first order derivatives can be resolved into potential, nonconservative position, gyroscopic, and dissipative forces.

The definition (8) of nonconservative position forces and the definition (10) of gyroscopic forces imply that the first must depend on coordinates q_k of the system, while the second depend on velocities $q_k^{\cdot}$. However, the general definition (8) of the nonconservative position forces does not exclude the possibility of these forces depending also on velocities $q_k^{\cdot}$ and time t. The gyroscopic and dissipative forces may, also, depend not only on velocities $q_k^{\cdot}$ but on coordinates q_k and time t, as well. Certain theorems that determine the stability properties of motion of a system on the basis of force structure which satisfy these general defintions are given in [4].

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ON CERTAIN INTEGRAL RELATIONSHIPS IN THE KINETIC THEORY OF GASES

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A method is proposed for the determination of certain moments of the Boltzmann collision integral, which appear in boundary problem solutions in the kinetic theory of gases, by expansion in the velocity half-space without actually calculating these [1], which makes it possible to establish definite relationships (including those derived earlier [2, 3]) between the moments.

The solution of boundary value problems of the kinetic theory of gases by the method of expansion in the velocity half-plane necessitates the determination of certain integrals that are moments of the Boltzmann collision integral, which many authors rightly consider to be the most laborious part of solving problems by this method. The step-by-step method of direct calculation of these moments was developed in [1]. It makes it possible, in principle, to solve a wide class of boundary value problems of the kinetic theory. In practice the application of that method [1] necessitates, however, very laborious calculations and does not provide means for checking the obtained results (errors